

A generalized screw dislocation in a piezoelectric trimaterial body

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Abstract. A generalized screw dislocation in the piezoelectric trimaterial with two parallel plane interfaces is studied. The Burgers vector of the generalized screw dislocation normal to the isotropic base plane and a discontinuous electric potential locates in the isotropic plane, simultaneously a line force and a line charge are acted at the core. The singular solution of a generalized screw dislocation in a homogeneous piezoelectric ceramic is used as the basis for solving the piezoelectric trimaterial body. A Schwarz-Neumann's alternating technique [1] is applied to construct this trimaterial solution. Further, the generalized Peach Koehler forces acting on the generalized screw dislocation are considered. Numerical illustrations for the interaction are given and some discussions on the effects of the material mismatches are given. It is shown that the present solutions are available and useful.

Keywords: Piezoelectric composite, dislocation, trimaterial body, alternating technique, complex potential

1. Introduction

Recently, piezoelectric thin film and layered structures have been increasingly adopted in the automobile components, opto-electronic devices and sensors, etc. due to the intrinsic coupling between electric and elastic behaviours. The defects, such as dislocations and micro-cracks, in these components are inevitable and affect the performance of the system. Therefore the reliability of these structures has attracted much attention.

The solution of a generalized screw dislocation in a component is very important to understand the mechanical behaviour of the material. The solution can enable us to describe the electric-elastic fields near the real dislocation and can also be used as a Green function to solve more complex problems. Moreover, the results of a generalized screw dislocation are qualitatively similar to that of a plane and general dislocation. Pak [2] first examined the generalized screw dislocation with elastic and electric-potential dislocations, and obtained the force on the dislocation in a piezoelectric material due to the presence of a free boundary. His results show that the electromechanical coupling coefficient has a significant influence on the electric-elastic fields near the generalized screw dislocation, and are very different from the fields in the pure elastic material. Liu et al. [3] studied the interaction between a generalized screw dislocation and an interface in a dissimilar piezoelectric body. They reveal that the interface may repeal the dislocation in materials with higher shear modulus due to the electromechanical coupling

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effect. Li and Duan [4] derived the interaction between a generalized screw dislocation and various free boundaries by using the conformal mapping technique. The closed-form solutions of electric-elastic fields for some special geometric configurations (wedge-shaped piezoelectric body, circular piezoelectric cylinder and piezoelectric plate) can be obtained. Recently, Chen et al. [5,6] dealt with the problems of a generalized screw dislocation in a piezoelectric bimaterial wedge by using a conformal mapping technique. However, all the above-mentioned research work focuses on the homogeneous or bimaterial piezoelectric components. To the authors' knowledge, there are no works that study the generalized screw dislocation in a trimaterial piezoelectric composite.

In this paper, we study the electroelastic problem of a generalized screw dislocation in the piezoelectric trimaterial body. The method is based on Neumann's modification of the Schwarz procedure, which was used in solving the elastic problem of a dislocation in an elastic trimaterial body [7,8]. Successive iterations are used to look for a series solution, which resembles the image method in potential theory with arranged infinite number of image singularities. This series solution provides a kernel function to solve the related crack problem in a piezoelectric trimaterial and the cumbersome and difficult inverse Fourier transform [9–12] can be avoided. Some numerical illustrations of the image force on the generalized screw dislocation due to the presence of an interface are given for various material mismatches. These results show that the image force depends on the electromechanical coupling factor, shear modulus ratio, piezoelectric coefficients ratio, and dielectric constants ratio.

2. Basic equations of the piezoelectric media

In this section, a transversely isotropic piezoelectric media with poling direction along the z -direction is considered. There are out-of-plane displacement $w(x, y)$ and in-plane electric potential $\phi(x, y)$, which are all independent on z . So the anti-plane governing equations for piezoelectric media can be written as [13]

$$\begin{aligned} c_{44}\nabla^2 w + e_{15}\nabla^2 \phi &= 0 \\ e_{15}\nabla^2 w - \varepsilon_{11}\nabla^2 \phi &= 0 \end{aligned} \quad (1)$$

where $c_{44}, e_{15}, \varepsilon_{11}$ are shear modulus, piezoelectric coefficient and dielectric constant, respectively, and $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplacian operator. Following Barnett and Lothe [14], the above equations can be rewritten as

$$\nabla^2 \mathbf{u} = 0, \quad \varepsilon_{11}e_{44} + e_{15}^2 \neq 0 \quad (2)$$

where

$$\mathbf{u} = \{w, \phi\}^T \quad (3)$$

is referred as the generalized displacement vector.

For linear piezoelectric material, the generalized strain (strain and electric field) can be given as

$$\begin{aligned} \varepsilon_x &= \{\gamma_{zx}, -E_x\}^T = \frac{\partial \mathbf{u}}{\partial x}, \\ \varepsilon_y &= \{\gamma_{zy}, -E_y\}^T = \frac{\partial \mathbf{u}}{\partial y}, \end{aligned} \quad (4)$$

and the generalized stress (stress and electric displacement) can be given as

$$\begin{aligned}\mathbf{t}_x &= \{\sigma_{zx}, D_x\}^T = \mathbf{C} \frac{\partial \mathbf{u}}{\partial x}, \\ \mathbf{t}_y &= \{\sigma_{zy}, D_y\}^T = \mathbf{C} \frac{\partial \mathbf{u}}{\partial y},\end{aligned}\tag{5}$$

where

$$\mathbf{C} = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix}\tag{6}$$

From Eq. (2), we know that \mathbf{u} is a harmonic vector function, which can be expressed by the real part of a vector complex potential with complex variable z , i.e.

$$\mathbf{u} = \text{Re}\{f_1(z), f_2(z)\}^T = \text{Re}[\mathbf{f}(z)], z = x + iy, \quad i = \sqrt{-1}\tag{7}$$

where Re denotes the real part. Substituting Eq. (7) into Eqs (4), (5), the generalized strain and stress can be expressed as

$$\begin{aligned}\varepsilon_x - i\varepsilon_y &= \mathbf{f}'(z) \\ \mathbf{t}_x - i\mathbf{t}_y &= \mathbf{C}\mathbf{f}'(z)\end{aligned}\tag{8}$$

where the prime is the derivative with respect to the argument z . Hence the resultant force and normal component of electric displacement along an arbitrary arc AB is

$$\mathbf{T} = \int_A^B \mathbf{t}_x dy - \mathbf{t}_y dx = \mathbf{C} \text{Im}[\mathbf{f}(z)]_A^B\tag{9}$$

where Im denotes the imaginary part and \mathbf{T} is the generalized traction.

3. A singularity in trimaterial component and the alternating technique

3.1. Singularity in a homogeneous piezoelectric body

Consider a generalized screw dislocation located at the point (x_0, y_0) in an infinite homogeneous piezoelectric body. The generalized dislocation has a Burgers vector normal to the isotropic base plane, a discontinuous electric potential across the slip plane, and a line force and a line charge at the core. Following Pak [2], the complex function can be expressed as

$$\mathbf{f}_0(z) = \mathbf{A} \ln(z - z_0), \quad z_0 = x_0 + iy_0\tag{10}$$

with

$$\mathbf{A} = \frac{1}{2\pi i}(-i\mathbf{C}^{-1}\mathbf{p} + \mathbf{b}), \quad \mathbf{p} = \{p, -q\}^T, \quad \mathbf{b} = \{b_z, b_\phi\}^T\tag{11}$$

where b_z, b_ϕ are mechanical and electric dislocations, respectively, and p, q are line force and line charge, respectively.

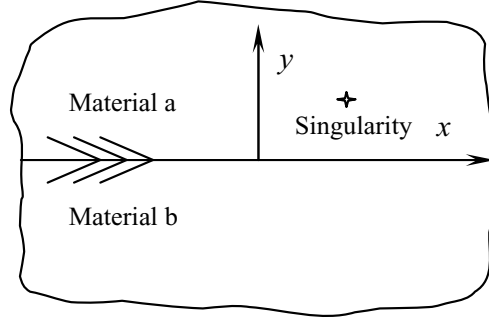


Fig. 1. A generalized screw dislocation in a bimaterial.

3.2. Singularity in a bimaterial body

The solution of a generalized screw dislocation in a bimaterial can be obtained from the solution in a homogeneous body by the technique of analytical continuation. Here the piezoelectric composite with two dissimilar transversely isotropic piezoelectric materials is considered (Fig. 1). It is assumed that material a occupies the upper half space and material b occupies the lower half space, and the interface is perfect. A generalized screw dislocation is located at material a , and we assume the complex potential functions is [15]

$$\mathbf{f}(z) = \begin{cases} \mathbf{f}_a(z) + \mathbf{f}_0(z) & z \in a \\ \mathbf{f}_b(z) & z \in b \end{cases} \quad (12)$$

where the subscripts a and b denote the material a and b , respectively, and $\mathbf{f}_a(z)$, $\mathbf{f}_b(z)$ are the analytical functions in the upper half space and lower half space, respectively. The generalized traction and generalized displacement are continuous along the interface of two materials due to the perfect bonding. From Eqs (7), (9) and (12), the following expressions are obtained

$$\begin{aligned} \mathbf{f}_a(x) + \overline{\mathbf{f}}_a(x) + \mathbf{f}_0(x) + \overline{\mathbf{f}}_0(x) &= \mathbf{f}_b(x) + \overline{\mathbf{f}}_b(x) \\ \mathbf{C}_a \mathbf{f}_a(x) - \mathbf{C}_a \overline{\mathbf{f}}_a(x) + \mathbf{C}_a \mathbf{f}_0(x) - \mathbf{C}_a \overline{\mathbf{f}}_0(x) &= \mathbf{C}_b \mathbf{f}_b(x) - \mathbf{C}_b \overline{\mathbf{f}}_b(x) \end{aligned} \quad (13)$$

Here $\overline{\mathbf{f}}_a(z)$, $\mathbf{f}_0(z)$ are analytical functions in the lower half space and $\overline{\mathbf{f}}_b(z)$, $\overline{\mathbf{f}}_0(z)$ are analytical functions in the upper half space. So using the method of analytical continuation, from Eq. (13) the complex potential functions can be expressed as

$$\begin{cases} \mathbf{f}_a(z) = \mathbf{V}_{ba} \overline{\mathbf{f}}_0(z) & z \in a \\ \mathbf{f}_b(z) = \mathbf{U}_{ba} \mathbf{f}_0(z) & z \in b \end{cases} \quad (14)$$

where $\overline{\mathbf{f}}_0(z) = \overline{\mathbf{f}_0(\bar{z})}$ and

$$\begin{aligned} \mathbf{V}_{ba} &= (\mathbf{C}_a + \mathbf{C}_b)^{-1}(\mathbf{C}_a - \mathbf{C}_b) \\ \mathbf{U}_{ba} &= 2(\mathbf{C}_a + \mathbf{C}_b)^{-1}\mathbf{C}_a \end{aligned} \quad (15)$$

Combining Eqs (12) and (14), the complete solution of a generalized screw dislocation located at the upper half space is obtained.

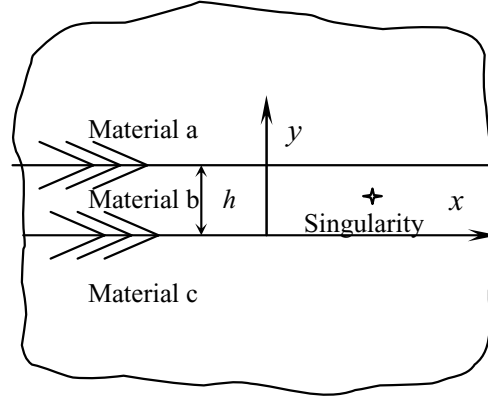


Fig. 2. A generalized screw dislocation in a trimaterial.

If the bonding interface of bimaterial is along the plane $y = h$, the Eq. (13) can be written as

$$\begin{aligned}
 & \mathbf{f}_a(x + ih) + \overline{\mathbf{f}}_a(x - ih) + \mathbf{f}_0(x + ih) + \overline{\mathbf{f}}_0(x - ih) \\
 &= \mathbf{f}_b(x + ih) + \overline{\mathbf{f}}_b(x - ih) \mathbf{C}_a \mathbf{f}_a(x + ih) - \mathbf{C}_a \overline{\mathbf{f}}_a(x - ih) + \mathbf{C}_a \mathbf{f}_0(x + ih) - \mathbf{C}_a \overline{\mathbf{f}}_0(x - ih) \\
 &= \mathbf{C}_b \mathbf{f}_b(x + ih) - \mathbf{C}_b \overline{\mathbf{f}}_b(x - ih)
 \end{aligned} \tag{16}$$

so the complex functions in Eq. (14) should be modified as

$$\begin{cases} \mathbf{f}_a(z) = \mathbf{V}_{ba} \overline{\mathbf{f}}_0(z - 2hi) & z \in a \\ \mathbf{f}_b(z) = \mathbf{U}_{ba} \mathbf{f}_0(z) & z \in b \end{cases} \tag{17}$$

These complex functions will be used in the next section.

3.3. The alternating technique

The alternating technique is used to solve the problem of a generalized screw dislocation in a piezo-electric trimaterial with two parallel interfaces at $y = 0$ and $y = h$ as shown in Fig. 2. The regions a , b and c are occupied by material a , b and c , respectively, and the perfect bonding is along the interfaces $y = 0$ and $y = h$. A generalized screw dislocation is located in material b . It is difficult to satisfy the continuity conditions along two interfaces at the same time. Using the alternating technique with following steps would solve the complete solution.

First step: let the material a be the same as the material b , which is bonded to material c along the interface $y = 0$. Like the procedure in Section 3.2, changing the notations the complex functions in this case can be expressed as

$$\begin{cases} \mathbf{f}_1(z) = \mathbf{V}_{cb} \overline{\mathbf{f}}_0(z) + \mathbf{f}_0(z) & z \in a \cup b \\ \mathbf{f}_{c0}(z) = \mathbf{U}_{cb} \mathbf{f}_0(z) & z \in c \end{cases} \tag{18}$$

and

$$\begin{aligned}
 \mathbf{V}_{cb} &= (\mathbf{C}_b + \mathbf{C}_c)^{-1} (\mathbf{C}_c - \mathbf{C}_b) \\
 \mathbf{U}_{cb} &= 2(\mathbf{C}_b + \mathbf{C}_c)^{-1} \mathbf{C}_b
 \end{aligned} \tag{19}$$

where $f_0(z)$ is the singular solution in a homogeneous medium as shown in Eq. (10), $f_1(z)$ and $f_{c0}(z)$ are solutions satisfying the continuity of the generalized traction and generalized displacement across the interface $y = 0$. Since this result is based on the assumption that material a and material b have been composed to the same material b , so these fields are not the exact solutions. However they can be considered as providing a first approximate solution.

Second step: Let the material c be the same as material b , which is bonded to material a along the interface $y = h$. $f_1(z)$ in Eq. (18) has a singularity in material $b \cup c$, and is treated as a homogeneous solution of material b . If we introduce functions $f_{a1}(z)$ and $f_{b1}(z)$ to satisfy the continuity conditions at the interface $y = h$, similar to Eq. (17) we will have

$$\begin{cases} f_{a1}(z) = U_{ab}f_1(z) & z \in a \\ f_{b1}(z) = V_{ab}\overline{f_1}(z - 2hi) & z \in b \cup c \end{cases} \quad (20)$$

where $f_1(z)$ is the function in Eq. (18). Since this result is based on the assumption that the material c is the same as material b , the fields corresponding $f_{a1}(z)$ and $f_{b1}(z)$ are also not the exact solution.

Third step: again consider the material a and b are composed of the same material b , which is bonded to material c along the interface $y = 0$. $f_{b1}(z)$ in Eq. (20) has a singularity in material $a \cup b$ and is treated as a homogeneous solution of material b . So we introduce functions $f_2(z)$ and $f_{c1}(z)$ to satisfy the continuity conditions at the interface $y = 0$. Following Eq. (14), we arrive at

$$\begin{cases} f_2(z) = V_{cb}\overline{f_{b1}}(z) & z \in a \cup b \\ f_{c1}(z) = U_{cb}f_{b1}(z) & z \in c \end{cases} \quad (21)$$

Since this result is based on the assumption that material a is made up of material b , the fields corresponding $f_2(z)$ and $f_{c1}(z)$ cannot be the exact solution.

Fourth step: repeat the second and third steps and obtain functions $f_{an}(z)$, $f_{bn}(z)$, $f_{cn}(z)$ and $f_{n+1}(z)$ ($n = 2, 3, \dots$), alternatively. Finally we can obtain the complex potential functions in material a , b and c as

$$f(z) = \begin{cases} f_{a1}(z) + f_{a2}(z) + f_{a3}(z) + \dots & z \in a \\ f_1(z) + f_{b1}(z) + f_2(z) + f_{b2}(z) + \dots & z \in b \\ f_{c0}(z) + f_{c1}(z) + f_{c2}(z) + \dots & z \in c \end{cases} \quad (22)$$

Substituting function $f_0(z)$ in Eq. (10) into Eq. (22), the above equations can be written in series forms, ie.

$$f(z) = \begin{cases} U_{ab} \sum_{n=1}^{\infty} f_n(z) & z \in a \\ \sum_{n=1}^{\infty} [f_n(z) + V_{ab}\overline{f_n}(z - 2hi)] & z \in b \\ U_{cb}f_0(z) + U_{cb}V_{ab} \sum_{n=1}^{\infty} \overline{f_n}(z - 2hi) & z \in c \end{cases} \quad (23)$$

where the recurrence formula for $f_n(z)$ is

$$f_{n+1}(z) = \begin{cases} f_0(z) + V_{cb}\overline{f_0}(z) & n = 0 \\ V_{cb}V_{ab}f_n(z + 2hi) & n = 1, 2, 3, \dots \end{cases} \quad (24)$$

If the generalized screw dislocation locates in the material c , the corresponding complex potential functions are expressed as

$$\mathbf{f}(z) = \begin{cases} \mathbf{U}_{ab} \sum_{n=1}^{\infty} \mathbf{f}_n(z) & z \in a \\ \sum_{n=1}^{\infty} [\mathbf{f}_n(z) + \mathbf{V}_{ab} \overline{\mathbf{f}}_n(z - 2hi)] & z \in b \\ \mathbf{f}_0(z) + \mathbf{V}_{bc} \overline{\mathbf{f}}_0(z) + \mathbf{U}_{cb} \mathbf{V}_{ab} \sum_{n=1}^{\infty} \overline{\mathbf{f}}_n(z - 2hi) & z \in c \end{cases} \quad (25)$$

in which the recurrence formula for $\mathbf{f}_n(z)$ is

$$\mathbf{f}_{n+1}(z) = \begin{cases} \mathbf{U}_{bc} \mathbf{f}_0(z) & n = 0 \\ \mathbf{V}_{cb} \mathbf{V}_{ab} \mathbf{f}_n(z + 2hi) & n = 1, 2, 3, \dots \end{cases} \quad (26)$$

3.4. Convergence of the series solution

The series solution obtained by the alternating technique can converge to a true solution, which is proved for isotropic elastic materials [1] and anisotropic elastic materials [7]. In series solution of our case, there are $|\mathbf{f}'_n(z + 2hi)| < |\mathbf{f}'_n(z)|$ for z in the region $a \cup b$ and $|\overline{\mathbf{f}}'_n(z - 2hi)| < |\overline{\mathbf{f}}'_n(z)|$ for z in the region $b \cup c$, in which $|\bullet|$ stands for the magnitude of a vector. So the complex functions have following relations

$$\begin{aligned} |\mathbf{f}'_{n+1}(z)| &\leq \|\mathbf{V}_{cb} \mathbf{V}_{ab}\| |\mathbf{f}'_n(z + 2hi)| < \|\mathbf{V}_{cb} \mathbf{V}_{ab}\| |\mathbf{f}'_n(z)|, \text{ for } z \text{ in the material } a \cup b \\ |\overline{\mathbf{f}}'_{n+1}(z)| &\leq \|\mathbf{V}_{cb} \mathbf{V}_{ab}\| |\overline{\mathbf{f}}'_n(z + 2hi)| < \|\mathbf{V}_{cb} \mathbf{V}_{ab}\| |\overline{\mathbf{f}}'_n(z)|, \text{ for } z \text{ in the material } b \cup c \end{aligned} \quad (27)$$

in which the matrix norm $\|a\|$ is defined as the largest magnitude of its eigenvalues. The convergence of the series solution for z in the arbitrary region is satisfied when the norm of $\|\mathbf{V}_{ab} \mathbf{V}_{cb}\|$ is less than or equal 1. Though these conditions cannot always be satisfied for arbitrary combination of transversely isotropic piezoelectric materials, but for most combinations of materials, this condition can be satisfied, which will be verified numerically in the next section. Moreover, the convergence rate is rather rapid and only four or five terms can give good approximation for more combinations of materials.

For the special case that material a (or c) is isotropic elastic materials, the norm of \mathbf{V}_{ab} (or \mathbf{V}_{cb}) is equal to 1, which is proved in Appendix A by solving the eigenvalues of \mathbf{V}_{ab} (or \mathbf{V}_{cb}). If material a (or c) is not existent or is an insulating rigid, the solutions are still valid and bimaterial matrices are

$$\mathbf{V}_{ab}(\mathbf{V}_{cb}) = \mathbf{I}, \text{ for } a \text{ (or } c) \text{ is empty} \quad (28a)$$

$$\mathbf{V}_{ab}(\mathbf{V}_{cb}) = -\mathbf{I}, \text{ for } a \text{ (or } c) \text{ is an insulating rigid material} \quad (28b)$$

3.5. Generalized Peach-Koehler force

Pak [2] defined the generalized Peach Koehler forces acting on a generalized screw dislocation as

$$\begin{aligned} F_x &= b_z \sigma_{zy}^0 + b_\phi D_y^0 + p \gamma_{zx}^0 + q E_x^0 \\ F_y &= -b_z \sigma_{zx}^0 - b_\phi D_x^0 + p \gamma_{zy}^0 + q E_y^0 \end{aligned} \quad (29)$$

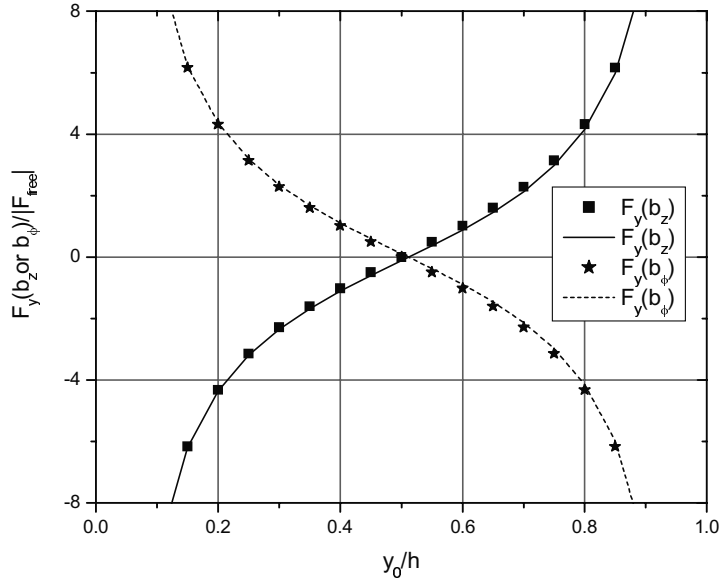


Fig. 3. Normalized image forces on a screw dislocation in a strip with free-free surfaces. The lines denote the present results and the marks denote the closed-form results.

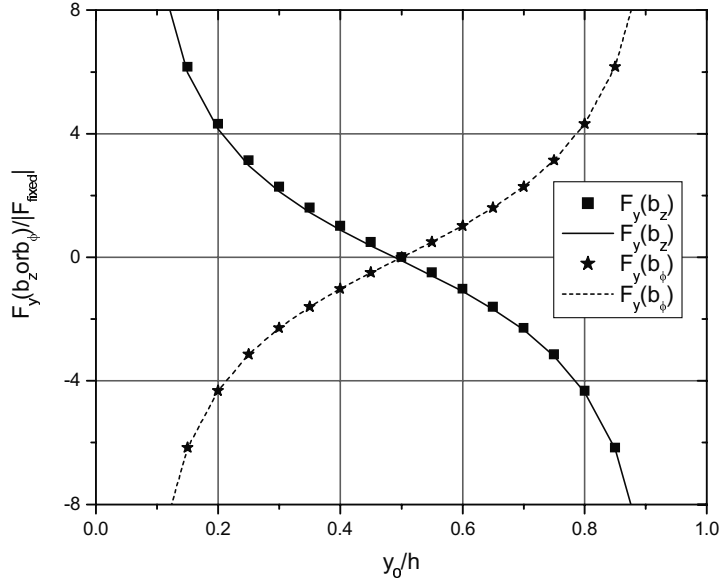


Fig. 4. Normalized image forces on a screw dislocation in a strip with fixed-fixed surfaces. The lines denote the present results and the marks denote the closed-form results.

where the variables $\sigma_{zx}^0, \sigma_{zy}^0, \gamma_{zx}^0, \gamma_{zy}^0, D_x^0, D_y^0, E_x^0$ and E_y^0 are

$$\begin{bmatrix} \gamma_{zx}^0 - i\gamma_{zy}^0 \\ -E_x^0 + iE_y^0 \end{bmatrix} = \mathbf{f}'^*(z_0) \quad (30a)$$

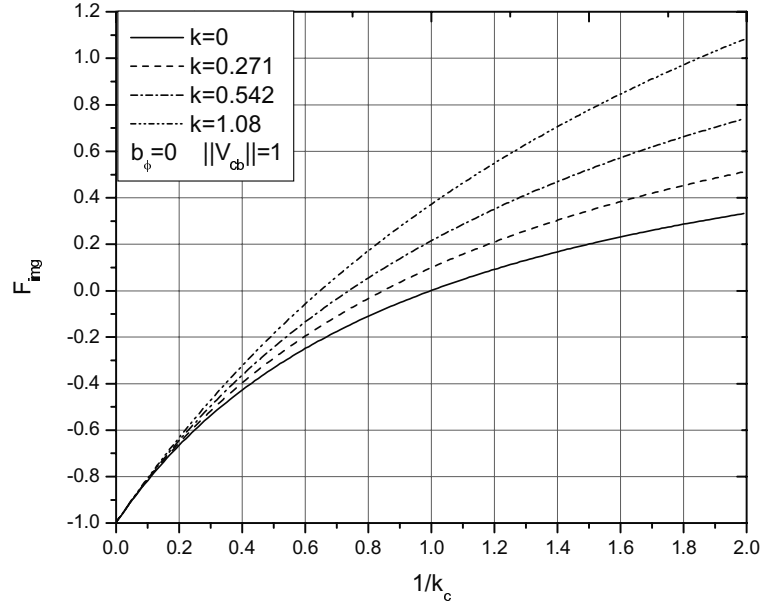


Fig. 5. Variation of the normalized image forces with shear modulus rate $1/k_c$.

$$\begin{bmatrix} \sigma_{zx}^0 - i\sigma_{zy}^0 \\ D_x^0 - iD_y^0 \end{bmatrix} = \mathbf{C}\mathbf{f}'^*(z_0) \quad (30b)$$

where function $\mathbf{f}^*(z)$ is the regular terms of complex function $\mathbf{f}(z)$, i.e.

$$\mathbf{f}^*(z) = \mathbf{V}_{cb}\bar{\mathbf{f}}_0(z) + \sum_{n=2}^{\infty} \mathbf{f}_n(z) + \sum_{n=1}^{\infty} [\mathbf{V}_{ab}\bar{\mathbf{f}}_n(z - 2hi)] \quad (31)$$

for a generalized screw dislocation in the material b in the trimaterial, and

$$\mathbf{f}^*(z) = \mathbf{V}_{bc}\bar{\mathbf{f}}_0(z) + \mathbf{U}_{cb}\mathbf{V}_{ab} \sum_{n=1}^{\infty} \bar{\mathbf{f}}_n(z - 2hi) \quad (32)$$

for a generalized screw dislocation in the material c in the trimaterial. Substituting Eqs (30) and (31) or (32) into Eq. (29), the image forces per unit length of a generalized screw dislocation due to two parallel interfaces in the trimaterial are

$$\begin{aligned} F_x &= 0 \\ F_y &= -\mathbf{b}^T \text{Re}[\mathbf{C}\mathbf{f}'^*(z_0)] - \mathbf{p}^T \text{Im}[\mathbf{f}'^*(z_0)] \end{aligned} \quad (33)$$

In order to reveal the interaction between a generalized screw dislocation and an interface or boundary, a line force and line charge are assumed to be zero in the next numerical examples. Here this generalized screw dislocation is called a piezoelectric screw dislocation.

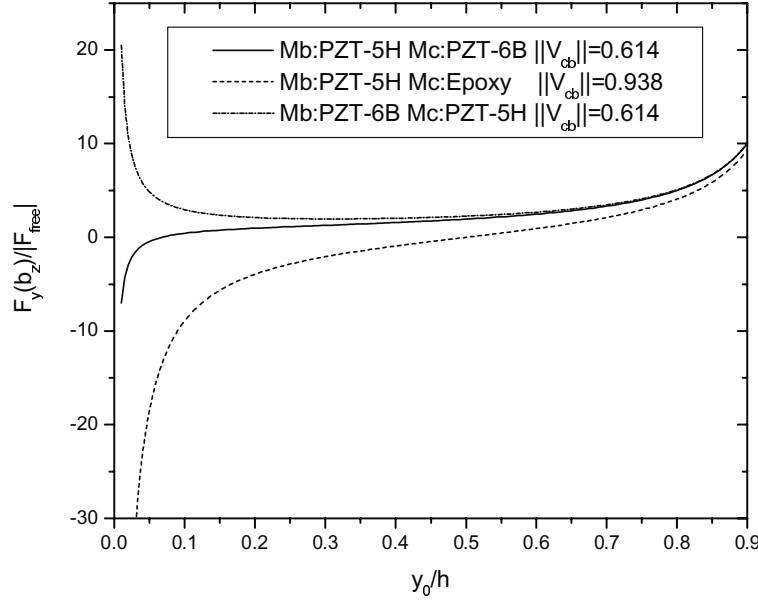


Fig. 6. The normalized image forces on a dislocation b_z vs. its position.

4. Numerical analysis and discussion

4.1. A piezoelectric screw dislocation in the homogeneous piezoelectric strip

To verify that the present solution is valid, a piezoelectric strip including a piezoelectric screw dislocation is considered. Comparing the closed-form solution obtained in Appendix B, Figs 3 and 4 give the normalized image forces acting on the mechanical (or electrical) dislocation with free-free surfaces and fixed-fixed surfaces, respectively. The normalized constant F_{free} (or F_{fixed}) is the image force on the piezoelectric screw dislocation at a distance h from the free (or fixed) surface in half-plane piezoelectric body. The figures show that the free surfaces attract the mechanical dislocation but repel the electrical dislocation. This means that the position in the middle of the strip is a stable point for electrical dislocation and a metastable point for mechanical dislocation. In contradistinction, Contrariwise, the fixed surfaces repel the mechanical dislocation and attract the electrical dislocation. This means that the position at the middle of strip is the metastable point for electrical dislocation and the stable point for mechanical dislocation. In contradistinction, the fixed surfaces repel the mechanical dislocation and attract the electric dislocation. This means that the position in the middle of the strip is a metastable point for the electric dislocation and a stable point for the mechanical dislocation. These results of the present model are in good agreement with the results of the closed-form solution.

4.2. A piezoelectric screw dislocation in a strip bonded to a half space

For film structures, the lattice misfit strain can be relaxed by the formation of dislocation depending on the thickness and growth conditions. The piezoelectric screw dislocation is analysed in this case due to the mathematical simplicity and the direct analogy to the edge dislocation problem. A piezoelectric screw dislocation in a finite strip or thin film (material b) bonded to a half space (material c), which is a special trimaterial when material a is assumed empty, is considered in this section. If we assume that the

Table 1
The material properties for transversely isotropic piezoelectric ceramics

Material	$c_{44} (\times 10^9 \text{N/m}^2)$	$e_{15} (\text{C/m}^2)$	$\varepsilon_{11} (\times 10^{-9} \text{C/Vm})$	k
EPOXY	1.07	0	5.0	0
PZT-6B	27.1	4.6	3.6	0.271
PZT-5H	35.3	17.0	15.1	0.542
PZT-4	25.6	13.44	6.0	1.08

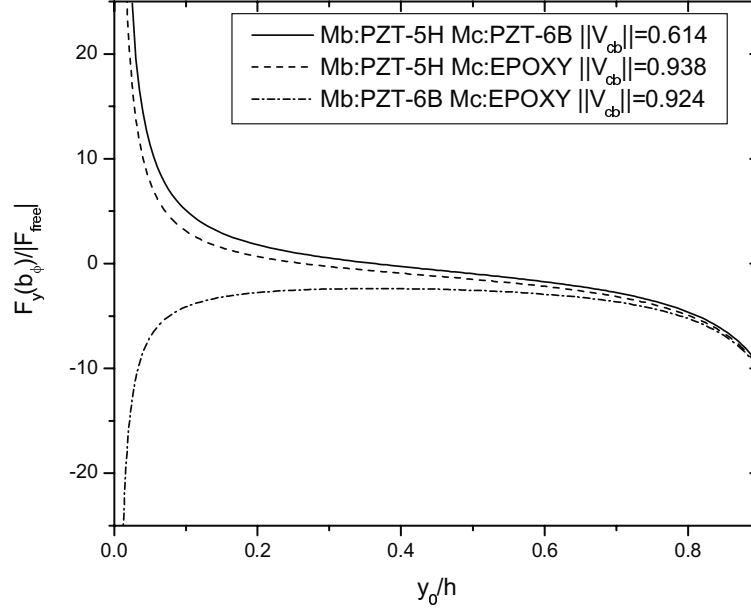


Fig. 7. The normalized image forces on a dislocation b_ϕ vs. its position.

piezoelectric screw dislocation is very close to the bimaterial interface in material b or the thickness h is rather large, then this model can reduce to the problem of a piezoelectric screw dislocation interacting with a piezoelectric bimaterial interface [3]. Figure 5 shows the variation of the normalized image forces on the dislocation b_z vs. shear modulus rate $k_c = c_{44}^b/c_{44}^c$, in which material c is pure elastic material. The normalized image force F_{img} is defined as

$$F_{img} = \frac{4\pi y_0}{c_{44}^b b_z^2} F_y \quad (34)$$

In this case $x_0 = 0$ and $y_0 = 0.1\%h$. The material properties of material b are listed in Table 1, in which $k = \sqrt{e_{15}^2/c_{44}\varepsilon_{11}}$ is the electromechanical coupling factor. Figure 5 indicates the present results are in good agreement with Fig. 2 in reference [3]. This also means that the present model is valid.

The four materials listed in Table 1 can compose some practical engineering pairs. In these material pairs, the norm of bimaterial matrix is less than 1 and the convergence of the series solutions can be satisfied. Figures 6, 7 give the normalized image forces on a dislocation b_z and b_ϕ vs. position y_0 , respectively. The numerical results show that the interface repels the dislocation b_z (b_ϕ) in the material with the lower shear modulus (in the higher dielectric constant) and attracts the dislocation b_z (b_ϕ) in the material with the higher shear modulus (in the lower dielectric constant), respectively. All the results will approach the same values when the dislocation is very close to the free-free surface.

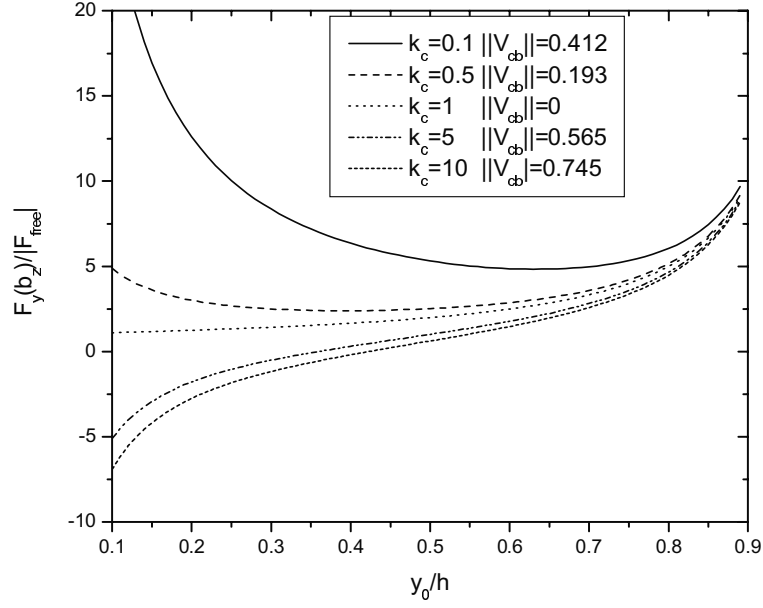


Fig. 8. The normalized image forces on a dislocation b_z vs. its position for different shear modulus ratios k_c of two dissimilar materials.

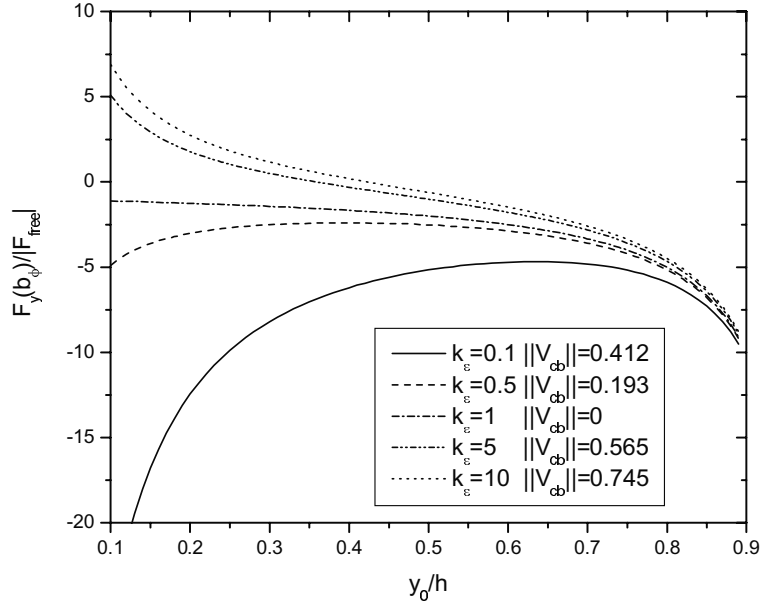


Fig. 9. The normalized image forces on a dislocation b_ϕ vs. its position for different dielectric ratios k_ϵ .

4.3. The influence of material mismatches on image forces

In order to investigate the effect of elastic, piezoelectric and dielectric mismatches on the image forces, we define the shear modulus, piezoelectric and dielectric ratios, respectively, i.e.

$$k_c = c_{44}^b/c_{44}^c, \quad k_e = e_{15}^b/e_{15}^c, \quad k_\epsilon = \epsilon_{11}^b/\epsilon_{11}^c \quad (35)$$

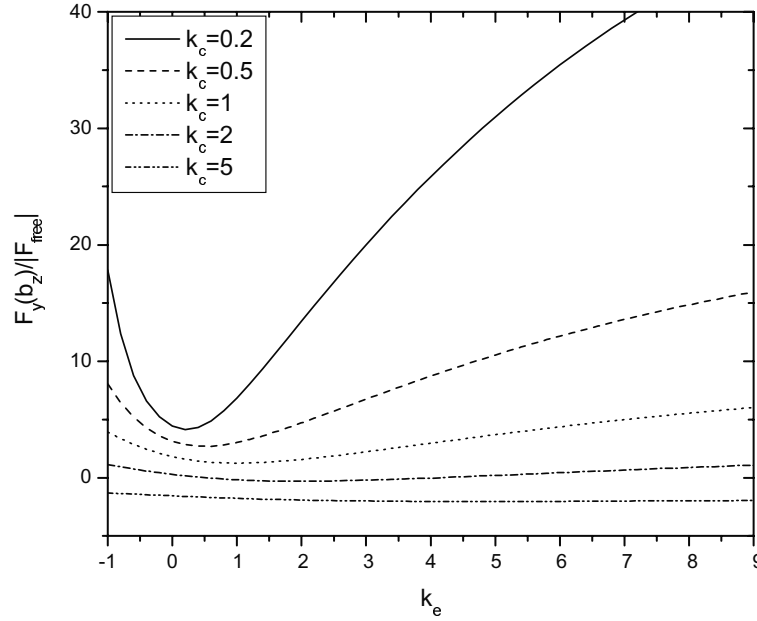


Fig. 10. Variations of the normalized forces on a dislocation b_z vs. the piezoelectric ratios k_e for different shear modulus ratios k_c .

In the following computation, one of the mismatches ratios varies keeping the other two equal to 1. Figures 8, 9 show the normalized image forces on b_z and b_ϕ vs. position y_0 with different material mismatches, respectively. In these figures, the norms of the bimaterial matrix are listed that are all less than 1. The image force on b_z increases with the elastic mismatches decreasing and the image force on b_ϕ increases with the dielectric mismatches increasing. Figures 10 and 11 show the normalized image force on b_z and b_ϕ close to the interface ($y_0 = 0.01h$) vary with the piezoelectric mismatch ratio for different shear modulus ratios and dielectric constant ratios, respectively. These two figures indicate that the interface repels (attracts) the dislocation b_z (or b_ϕ) in the material with lower shear modulus (or lower dielectric constant). Further it is seen that when a dislocation b_z (or b_ϕ) is in the material with a higher shear modulus (or higher dielectric constant), the interface may also repel (or attract) the dislocation b_z (or b_ϕ), which depend on the piezoelectric ratio of two materials and the position of the dislocation in strip due to the presence of free surface. In all cases, the norm of the bimaterial matrix is less than 1, which insure the convergence of series solutions.

5. Conclusions

The problem of a generalized screw dislocation in the trimaterial piezoelectric component is firstly analysed by applying the alternating technique and the method of analytic continuation. The singular solution of a generalized screw dislocation in homogeneous bodies is considered as the basic solution to derive the trimaterial solution with the same singularities in a series form. The norm of the bimaterial matrix is less than or equal to 1 for most combinations of materials, which insure the convergence of the trimaterial solution. The present solutions can provide useful Green functions for the solutions of corresponding crack problems. Further the generalized Peach Koehler forces on the generalized

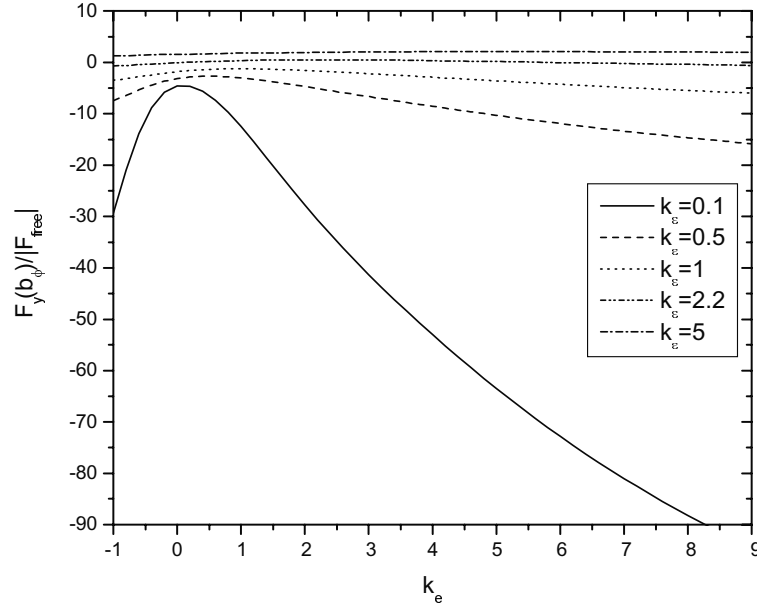


Fig. 11. Variations of the normalized forces on a dislocation b_ϕ vs. the piezoelectric ratios k_e for different dielectric ratios k_ε .

screw dislocation are obtained in series form. Some numerical examples are given, e.g. a finite strip, a bimaterial and a finite strip or thin film on semi-infinite substrate, which are all special cases of the trimaterial component. The results show that the different parameters such as dislocation location, coupling factor and material mismatches have an influence on the image force acting on the dislocation. The present results are in good agreement with that in the open literatures.

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Appendix A. Proof of $||\mathbf{V}_{ab}|| = 1$

Assume material a is an isotropic elastic material with shear modulus μ and material property matrix of material b is defined as

$$\mathbf{C}_b = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix} \quad (\text{A.1})$$

From Eq. (15) the bimaterial matrix \mathbf{V}_{ab} can be written as

$$\mathbf{V}_{ab} = \frac{1}{e_{15}^2 + \varepsilon_{11}(c_{44} + \mu)} \begin{bmatrix} \varepsilon_{11} & e_{15} \\ e_{15} & -(c_{44} + \mu) \end{bmatrix} \begin{bmatrix} c_{44} - \mu & e_{15} \\ e_{15} & -\varepsilon_{11} \end{bmatrix}$$

$$= \frac{1}{e_{15}^2 + \varepsilon_{11}(c_{44} + \mu)} \begin{bmatrix} e_{15}^2 + \varepsilon_{11}(c_{44} - \mu) & 0 \\ -2e_{15}\mu & e_{15}^2 + \varepsilon_{11}(c_{44} + \mu) \end{bmatrix} \quad (\text{A.2})$$

So the norm of the matrix \mathbf{V}_{ab} is defined as

$$\|\mathbf{V}_{ab}\| = \text{Max}(|\lambda_1|, |\lambda_2|) \quad (\text{A.3})$$

Using Eq. (A.2), it is obvious that $\|\mathbf{V}_{ab}\| = 1$ for arbitrary positive shear modulus μ .

Appendix B. Closed-form solution for transversely isotropic piezoelectric strip

Consider a generalized screw dislocation at (x_0, y_0) in a transversely isotropic piezoelectric strip, which has the generalized tractions free boundary conditions (free-free surfaces) and generalized displacements fixed boundary conditions (fixed-fixed surface), respectively. The conformal mapping,

$$\zeta = e^{\frac{\pi}{h}z}, \quad \zeta = \xi + i\eta \quad (\text{B.1})$$

will map the strip onto the upper half plane with corresponding boundary conditions. The complex function in a homogeneous medium is expressed as

$$f_0(\zeta) = \mathbf{A} \ln(\zeta - \zeta_0), \quad \zeta_0 = e^{\frac{\pi}{h}z_0}, \quad \zeta_0 = \xi_0 + i\eta_0 \quad (\text{B.2})$$

where matrix \mathbf{A} and z_0 are given in Eqs (11) and (10), respectively. By applying the method of analytic continuation, the solution form for a generalized screw dislocation at (ξ_0, η_0) in the upper half-plane is

$$f(\zeta) = \mathbf{A} \ln(\zeta - \zeta_0) + \overline{\mathbf{A}} \ln(\zeta - \overline{\zeta_0}) \quad (\text{B.3})$$

$$f(\zeta) = \mathbf{A} \ln(\zeta - \zeta_0) - \overline{\mathbf{A}} \ln(\zeta - \overline{\zeta_0}) \quad (\text{B.4})$$

where Eq. (B.3) corresponds to free-free surfaces and Eq. (B.4) corresponds to fixed-fixed surfaces. Substituting the expressions of ζ and ζ_0 into Eq. (B.3) or Eq. (B.4), the complete complex function $f(z)$ for the piezoelectric strip will be obtained.

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